Recent Advances in Agglomerated Multigrid

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Purpose

Improve efficiency of Reynolds-Averaged Navier-Stokes (RANS) simulations for complex-geometry and complex-flow applications

- Unstructured compressible flow method (FUN3D)
- Spalart-Allmaras 1-equation turbulence model
- Multigrid with agglomerated coarse grids
Previous Work

• Agglomerated multigrid methods well suited to unstructured grids
  – Nonlinear multigrid from 1977
  – Agglomeration methods from 1987
  – Speedups observed but gains fall short of gains for inviscid or laminar flows

• Current approach extends hierarchical multigrid method (1999) to unstructured grids
  – Assessed defect correction for compressible Euler (2010)
  – Developed agglomeration method preserving features of geometry (2010)
  – Critically assessed multigrid for diffusion (2010), identified by Venkatakrishnan (1996) as weakest part of agglomeration
  – Applied to complex inviscid/laminar/turbulent flows (2010/2011)
Multigrid for Euler (2011)

NACA 0012 airfoil; M= 0.5; Alpha =1.25

Fast convergence on all 5 grids

Convergence below discretization errors after one cycle on each grid

Vassberg and Jameson

Full Multigrid Cycle
Multigrid for Diffusion (2010)

DLR F6 Wing-Body Grids
Multigrid for Diffusion (2010)

DLR F6 Wing-Body

Convergence 50 times faster with multigrid
- Consistent coarse grid discretization
- Prismatic layers
- Line implicit solves and coarsening
3D Agglomerated Multigrid (2011)

Subsonic Laminar
20 Million nodes

Hemisphere Cylinder

Transonic RANS
12 Million nodes

Wing Alone

Computer Time

Continuity Residual

Single Grid
Multigrid

Turbulence Residual

Single Grid
Multigrid
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- Overall status
  - Uniformly successful for laminar and inviscid simulations
  - Limited success for RANS simulations
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Solving turbulence equations to zero residuals is one such question
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  Single grid

“When there is a difficult question that you cannot answer, there is also a simpler question that you cannot answer” (Achi Brandt)

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Why Such a Difficult Question?
Spalart-Allmaras Model
with negative turbulence variable provisions (2012)

- Nonlinear diffusion and source terms
- Production source terms associated with exponentially growing solutions are eventually balanced by other terms
  - Reduce diagonal positivity
  - Require gradients of mean flow variables
- RANS necessitates high aspect ratio, highly stretched grids
  - 3D grid refinements stretch computer resources
  - Questionable accuracy on coarse grids
- Discrete solutions depart from positivity of differential equations (provisions date to 1999)
  - Negative turbulence variable => zero eddy viscosity
  - Steady 3D flows with negative turbulence variable in far wake regions
  - Some first-order 3D cases are harder to solve than second-order
Remainder of Talk

- Hierarchical solver with adaptive time step
- Agglomerated and structured multigrid comparisons
- Transonic 3D wing-body computations
- Concluding remarks
Contributions of this Paper

- Improved hierarchical solver with adaptive pseudo-time step
- Assessment of current technology with systematic tests
  - Increasing complexity and grid refinement
  - Structured and agglomerated grids
- Parallelization improvements
- Eliminated degenerate least-square stencils (fine/coarse grids)
- Term-by-term formation of Jacobians
- Improved discretization in line-implicit regions
Hierarchical Solver

Nonlinear Multigrid

Relaxation
(Jacobian-Free Newton-Krylov)

Preconditioner
Nonlinear Multigrid

Full Approximation Scheme $V(\nu_1, \nu_2)$ Cycle

2-Level Cycle

$\nu_1$ Relaxations \hspace{2cm} $\nu_2$ Relaxations

**Fine Grid:**

Restriction \hspace{2cm} Prolongation

**Coarse Grid:**

- Recursive application of 2-grid cycle
- Mean flow and turbulence relaxed at every level
  - Loosely-coupled (meanflow, then turbulence)
  - Tightly-coupled
Jacobian-Free Newton Krylov

Generalized Conjugate Residual (GCR)

Target update equation is full linearization with adaptive time step (CFL)

\[
\left( \frac{V}{\Delta \tau} + \frac{\partial R}{\partial Q} \right) \delta Q = -R \quad ; \quad Q = Q + \delta Q
\]

\( V \equiv \) volume \quad \( R \equiv \) residual

\( \Delta \tau \equiv \) time step \quad \( Q \equiv \) solution

Full linearization is through Jacobian-free matrix evaluation

• Real-valued (uncertainties from round-off and constrained growth)
• Complex-valued (nominally exact)

GCR combines preconditioner directions to minimize residual, generally with loose tolerance and a few projections

\[
\left\| \left( \frac{V}{\Delta \tau} + \frac{\partial R}{\partial Q} \right) \delta Q + R \right\| \leq f \| R \| \quad 0.5 \leq f \leq 0.98
\]

GCR often stabilizes divergent preconditioner subiterations
Preconditioner

Jacobian Approximations with Subiterations

Target update (direction) equation is approximate linearization with adaptive time step

\[
\left( \frac{V}{\Delta \tau} + \frac{\partial R}{\partial Q} \right) \delta Q = -R
\]

where linearization is approximate, e.g., on primal grids:
• first-order accurate inviscid terms (defect correction)
• exact viscous terms

Alternating multicolor point-implicit and line-implicit subiterations, solving with loose tolerance

\[
\left\| \left( \frac{V}{\Delta \tau} + \frac{\partial R}{\partial Q} \right) \delta Q + R \right\| \leq f \| R \| \quad 0.5 \leq f \leq 0.98
\]

Can often take many subiterations to meet even minimal tolerance of \( f \leq 0.98 \)
Pseudo-Time Step (CFL) Adaptation

• Motivated by recent work of Allmaras et al. (2011) on robust Newton solver
  – Direct linear solver
  – CFL low in highly nonlinear regions and where allowable changes exceeded
  – CFL eventually high with quadratic convergence

• Current approach evolving
  – CFL reduced whenever linear systems are having difficulty reaching loose tolerance with minimal GCR projections
  – CFL reduced and null update nulled whenever linear systems converging but update extremely large
  – Otherwise, similar to Allmaras et al. but without quadratic convergence
Evolution of Target CFL Strategy
## Evolution of Target CFL Strategy

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<th>Preconditioner</th>
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Enabled by GCR
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<tr>
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<td>10,000</td>
<td>Consistent Defect Correction</td>
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</tbody>
</table>

- Enabled by CFL adaptation
- Enabled by GCR
## Turbulent Test Cases (SpeedUp over Single Grid)

V(3,3) ; CFL=200 (Quarter Horse CFL Target)

Based on Convergence to Machine Zero Residuals

<table>
<thead>
<tr>
<th>Geometry</th>
<th>Finest Grid Nodes</th>
<th>Agglomerated Multigrid SpeedUp</th>
</tr>
</thead>
<tbody>
<tr>
<td>2D Bump in a Channel</td>
<td>4K</td>
<td>3x</td>
</tr>
<tr>
<td>2D RAE Airfoil</td>
<td>98K</td>
<td>3x</td>
</tr>
<tr>
<td>2D Flat Plate</td>
<td>209K</td>
<td>9x</td>
</tr>
<tr>
<td>2D NACA 0012 Airfoil</td>
<td>919K</td>
<td>8x</td>
</tr>
<tr>
<td>2D Hemisphere Cylinder</td>
<td>960K</td>
<td>16x</td>
</tr>
<tr>
<td>3D Hemisphere Cylinder</td>
<td>15M</td>
<td>19x</td>
</tr>
<tr>
<td>3D Wing-Body-Tail (DPW4)</td>
<td>10M</td>
<td>7x</td>
</tr>
<tr>
<td>3D Wing-Body (DPW5)</td>
<td>15M</td>
<td>&lt; 1x</td>
</tr>
</tbody>
</table>
From Quarter Horse to Thoroughbred
In Target CFL
NACA 0012; M=0.15; Alpha = 15
Cycles to Machine Zero Residuals with Full Multigrid Cycle

<table>
<thead>
<tr>
<th>Grid Density</th>
<th>Agglomerated Multigrid V(3,3) Cycles CFL=200</th>
<th>Structured Multigrid V(2,2) Cycles CFL=10,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grid 1 (Fine)</td>
<td>276</td>
<td>24</td>
</tr>
<tr>
<td>Grid 2 (Medium)</td>
<td>241</td>
<td>23</td>
</tr>
<tr>
<td>Grid 3 (Coarse)</td>
<td>216</td>
<td>24</td>
</tr>
</tbody>
</table>
Hybrid unstructured grid
- prismatic near-field
- tetrahedral far-field

<table>
<thead>
<tr>
<th>Structured Grid Designation</th>
<th>Nodes</th>
</tr>
</thead>
<tbody>
<tr>
<td>L1</td>
<td>0.7 Million</td>
</tr>
<tr>
<td>L2</td>
<td>2.2 Million</td>
</tr>
<tr>
<td>L3</td>
<td>5.2 Million</td>
</tr>
</tbody>
</table>
Transonic Wing-Body
Structured V(2,2) Multigrid Cycle; CFL=10,000
M=0.85 ; Alpha Continued [1 to 3]
Transonic Wing-Body
M=0.85 ; Alpha Continued [1 to 4]

With grid-refinement for alpha > 3
• Lift < experiment
• No improvement with multigrid
Transonic Wing-Body

M=0.85; Alpha Continued [1 to 4], then decreasing

With grid-refinement, strong hysteresis (not observed in experiment)
Pressure Contours and Streamlines

Higher Lift
Alpha=2.25

Lower Lift
Alpha=2.25

Higher Lift
Alpha=2.25
Concluding Remarks

• Hierarchical solver with adaptive time step control proven useful for turbulent flows
  – Single grid and multigrid methods
  – Used in comparisons/adaptation of Park (Monday AM)
• Agglomeration multigrid assessed over range of tests
  – V(3,3) cycle with CFL=200
  – Comparable performance with structured-grid multigrid
  – Substantial improvement over single grid method
• Structured multigrid assessed over smaller range of tests
  – V(2,2) with CFL=10,000
  – Fast convergence for 2D and 3D comparable to inviscid
Future Research

• Single grid solver
  – Refinements in adaptation strategy to reduce sensitivity to selectable parameters (robust controller)

• Multigrid
  – Assess agglomeration multigrid with higher CFL numbers
  – Understand limitations observed for DPW5 grids
  – Apply ideal multigrid tools to assess where further inroads are possible
    • Ideal relaxation (tests coarse grid correction)
    • Ideal coarse grid (tests relaxation)
Research Possible through the NASA Fundamental Aeronautics Program

Support of cross-cutting technology development
- Peter Coen (Supersonics)
- Mike Rogers (Subsonic Fixed Wing)
- Susan Gorton (Rotorcraft)
- Mujeeb Malik (Revolutionary Computational Aerosciences)

First two authors supported by NASA contracts
- NNL12AB00T “Improvements of Unstructured Finite-Volume Solutions for Turbulent Flows”
- NNL09AA00A “Efficient Iterative Solutions for Turbulent Flows”
BackUp Slides Follow
Cycles (Single Grid; No Coarse Grid Correction)

Residual

Meanflow Residual
Turbulence Residual
Lift Coefficient

Meanflow Residual
Turbulence Residual
Lift Coefficient

Cycles (Single Grid; No Coarse Grid Correction)
## Coarse Grids: AgMG vs StMG

<table>
<thead>
<tr>
<th>Property</th>
<th>Agglomerated</th>
<th>Structured</th>
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<tbody>
<tr>
<td>Coarsening</td>
<td>Hierarchical (3-level)</td>
<td>Full (all levels)</td>
</tr>
<tr>
<td>Accuracy</td>
<td>First Order Inviscid, Second Order Viscous</td>
<td>Second Order Inviscid, Second Order Viscous</td>
</tr>
<tr>
<td>Jacobians</td>
<td>Approximate Viscous</td>
<td>Approximate Inviscid</td>
</tr>
<tr>
<td>Restriction</td>
<td>Conservative with Residual Averaging</td>
<td>Full Weighting (prolongation transpose)</td>
</tr>
<tr>
<td>Prolongation</td>
<td>Linear --&gt; Constant (viscous curved)</td>
<td>Linear (structured mapping)</td>
</tr>
<tr>
<td>Cycle / CFL</td>
<td>V(3,3) / CFL=200</td>
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NACA 0012 Airfoil
M=0.15; Finest Grid (919K)
Alpha Continued From [10 to 19] then [19 to -5] then [-5 to -3]

No evidence of hysteresis in computation
NACA 0012 Airfoil

M=0.15; Finest Grid (919K)
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O(10) improvement with multigrid
O(10) more cycles near zero lift