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TOWARDS RUBUST, HIGH ORDER AND ENTROPY STABLE ALGORITHMS FOR THE SOLUTION OF THE COMPRESSIBLE NAVIER-STOKES EQUATIONS ON UNSTRUCTURED GRIDS

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Although the formulation of new spatially high order schemes has received a boost in recent years, most high order techniques experience a loss of robustness when the solution contains discontinuities or even when physical features are under-resolved. Unfortunately, the variety of mathematically rigorous stabilization techniques developed for second-order accurate methods cannot be easily generalized to high order formulations. Only recently, a general procedure for developing stable finite difference schemes that satisfy a mathematical entropy inequality has been introduced by Fisher and Carpenter[1] in the context of summation-by-parts (SBP) operators and simultaneous-approximation-term (SAT). Entropy stability (SS) guarantees that the thermodynamic entropy is bounded for all time in L_2, provided that density and temperature remain positive and boundary data is well-posed. An entropy estimate is preserved via a telescoping cancellation of terms through the domain when using SBP-SAT operators and an entropy consistent numerical flux. In this work, recent developments of discontinous spectral methods with staggered-grid operators of arbitrary order and an implicit time-stepping algorithm will be presented. Discontinuous interfaces are used between elements and tensor product arithmetic extends the new formulation to the three-dimensional compressible Navier-Stokes equations on unstructured grids. An entropy stable reconstructed flux is used for the inviscid interface coupling between elements, whereas a local discontinuous Galerkin approach is employed for the viscous terms.

In this work, the high-order implicit-explicit Runge–Kutta (IMEX-RK) method of Kennedy and Carpenter [2] is implemented to partially overcome geometric-induced stiffness. The nonlinear system of ordinary differential equations is inverted efficiently using a Newton-GMRES algorithm preconditioned by an approximation of the analytical Jacobian matrix. Such a matrix is computed efficiently using matrix-matrix multiplies in compressed sparse row (CSR) format. Furthermore, a proportional-integral-derivative (PID) algorithm is implemented to allow automatic error-based time-step control.

Here, the convergence rate of the proposed entropy stable scheme is presented for the two-dimensional viscous shock. A nested sequence of random quadrilateral grids is used. (A random initial grid on a unit square is duplicated in x and y, then rescaled in size. The process is repeated several times to generate the sequence.) The shock profile is initially located in the middle of the domain and is simulated until $t = 1.00$. The Reynolds number is $Re = 10$ and the reference Mach number is $M = 2.5$. The perfect gas thermodynamic relation and a heat capacity ratio of 1.4 are used to close the system of equations. Errors for a random grid convergence study are shown in Table 1. Design order convergence (p+1) is observed for all orders for this two-dimensional Navier-Stokes test case[1]. Convergence rates when using collocated solution and flux points degenerated back to order p for this sequence of random grids.

Table 1: Error and convergence rate is shown for the Viscous Shock on random grids.

p=4	L_2 error	L_2 rate	L_∞ error	L_∞ rate
9x9	4.098E-05		9.090E-04	
17x17	1.916E-06	-4.418	6.375E-05	-3.833
33x33	6.464E-08	-4.889	2.061E-06	-4.950
65x65	2.114E-09	-4.933	9.808E-08	-4.393
129x129	6.710E-11	-4.978	2.569E-09	-5.254
p=5				
9x9	6.146E-06		1.515E-04	
17x17	1.256E-07	-5.612	5.706E-06	-4.730
33x33	2.116E-09	-5.891	1.340E-07	-5.412
65x65	3.437E-11	-5.944	1.333E-09	-6.651
p=6				
9x9	8.787E-07		2.407E-05	
17x17	8.778E-09	-6.645	3.286E-07	-6.195
33x33	8.032E-11	-6.772	3.000E-09	-6.797
65x65	8.619E-13	-6.542	4.688E-11	-5.977

The full study will include a detailed description of the implicit entropy stable algorithm for the three-dimensional Navier-Stokes equations. The benckmark test case used to highlight the outstanding properties of the solver is the compressible turbulent flow past a three-dimensional rod at $Re = 1 \times 10^7$. Challenges and difficulties that arise from this new class of solvers will also be discussed.

REFERENCES

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[1] p is the degree of the polynomial approximation of the solution in one cell.